Strings, Pulleys, and Centripetal Forces

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Atwood Machines

Definition An Atwood Machine is a system where there are two masses, m_1 and m_2, that are connected via string over a pulley which are both massless and frictionless.

Acceleration Calculating the acceleration of an Atwood Machine is simple enough. First, let g equal the acceleration of gravity. Since each mass pulls directly downward on either side of the pulley, their forces exactly oppose one another. Consequently,

$$F_{\text{net}} = m_1 g - m_2 g$$

Note the sign of F_{net} will determine the direction of acceleration. If $F_{\text{net}} \geq 0$, then the acceleration will pull the mass m_1 downward. Else if $F_{\text{net}} < 0$, then the acceleration will pull the mass m_2 downward. Indeed,

$$\begin{aligned}
(m_1 + m_2)a &= F_{\text{net}} \\
&= \frac{F_{\text{net}}}{m_1 + m_2} \\
&= \frac{m_1 g - m_2 g}{m_1 + m_2} \\
a &= g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)
\end{aligned}$$

Tension To calculate the string's tension, we can use one of the force equations obtained by realizing that the difference in force exerted between the mass and the system's tension is equivalent to the force of accelerating the mass at the system's acceleration.

$$\begin{aligned}
m_1 g - T &= m_1 a \\
T &= m_1 g - m_1 a
\end{aligned}$$

Substitute previously found a,

$$
\begin{aligned}
&= m_1 g - m_1 g \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \\
&\vdots \\
&= \frac{2 g m_1 m_2}{m_1 + m_2} \\
T &= \frac{2g}{m_1^{-1} + m_2^{-1}}
\end{aligned}
$$

Centripetal Forces

Centripetal generally means moving toward a center. This is essential to understanding centripetal forces.

Centripetal Acceleration As the proof is too high-level to be described here,[1] the centripetal acceleration formula is provided as follows for *Uniform Circular Motion*:[2]

$$a_c = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}\theta} \cdot \frac{\mathrm{d}\theta}{\mathrm{d}t} = v \cdot \frac{v}{r} = \frac{v^2}{r}$$

The force counterpart is F_c such that,

$$F_c = m a_c = \frac{m v^2}{r}$$

Average Velocity To calculate the average velocity—also termed *tangential velocity*—of an object in UCM, one may determine the distance traveled (number of revolutions R multiplied by the circumference of each revolution c) and divide that by the total time taken Δt.

$$\bar{v} = \frac{Rc}{\Delta t}$$

And consequently the instantaneous velocity,

$$v = c \cdot \frac{\mathrm{d}R}{\mathrm{d}t}$$

The circumference was factored out because it does not change with respect to time.

[1] Deriving the Centripetal Acceleration Formula, Matthew Van Eerde

[2] The given object is moving at a constant speed and along a circular path.